2.1 Introduction To Computer-Aided Learning In Mathematics

Computer-Aided Learning is widely used in the discipline of Mathematics. A working group in 1984 highlighted advantages in using computers, especially within the field of mathematical teaching. "Computers have some essential advantages over human beings – speed, accuracy and reliability in computation and display, and the capacity to handle complexity in either" [Burn et al (1998)]. It can be obviously seen that in mathematics the need for accuracy and reliability is imperative, and that the use of computers can therefore be extremely beneficial.

In most areas of teaching via CAL, information is input via a programmer and then viewed by a student. When using CAL in mathematical teaching there is also a need for displaying specialist information and symbols on the screen [Beevers et al (1991)]. There needs to be the ability therefore within the Mathematical CAL package to allow mathematical symbols and information to be presented for the user, and also if required to be re-entered by the user.

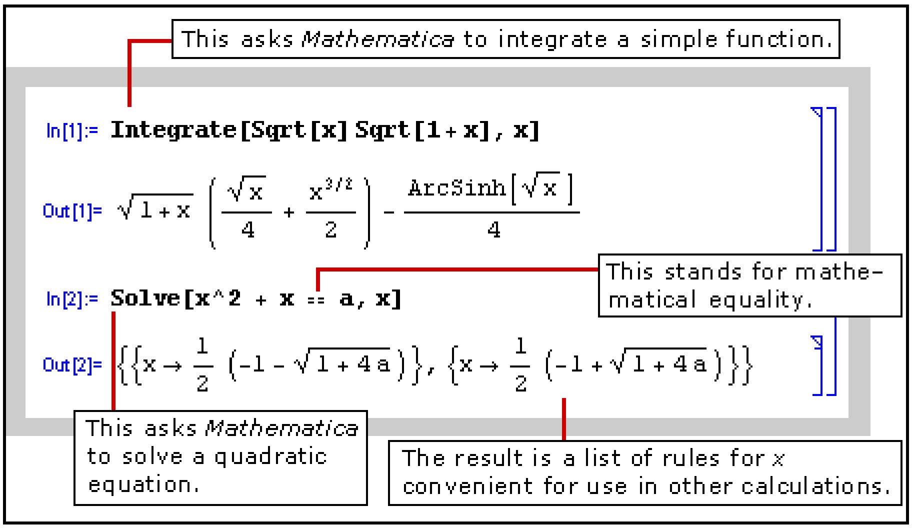
In Mathematics there are many correct forms in which an answer to a question can be laid out. Software Tools for CAL in Mathematics gives the following example "What is the derivative of x^2?" $2x$, or $x + x$, or even $(1 + 1)\, x$. The student would be correct in entering any of these answers. One view is that there is a need therefore that the computer will accept all possible permutations of the correct answer to a question. From an different viewpoint however it could be said that a student working at this level of mathematics should be expected to input $2x$ and therefore should be marked down or even wrong for inputting an alternative.

2.2 Review of current CAL resources

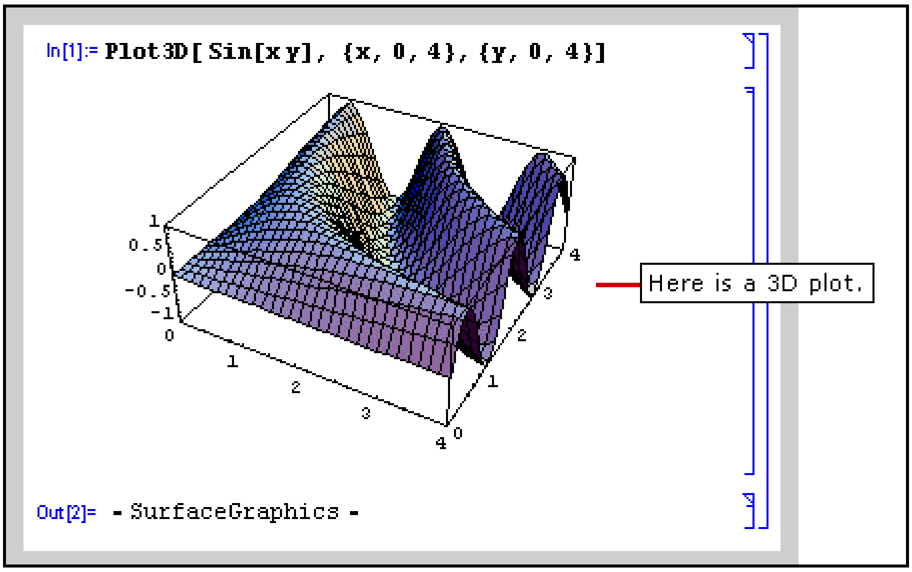
Computer Aided Learning takes many guises within Mathematics, for example using complex symbolic manipulation and graphing techniques with Mathematica software, or in handling vast amounts of data in the forms of spreadsheets and databases, for instance Microsoft Excel or Access. However, a common form of Computer Aided Learning comes in the form of teaching and assessment packages (CAA).

2.2.1 Generic Tools

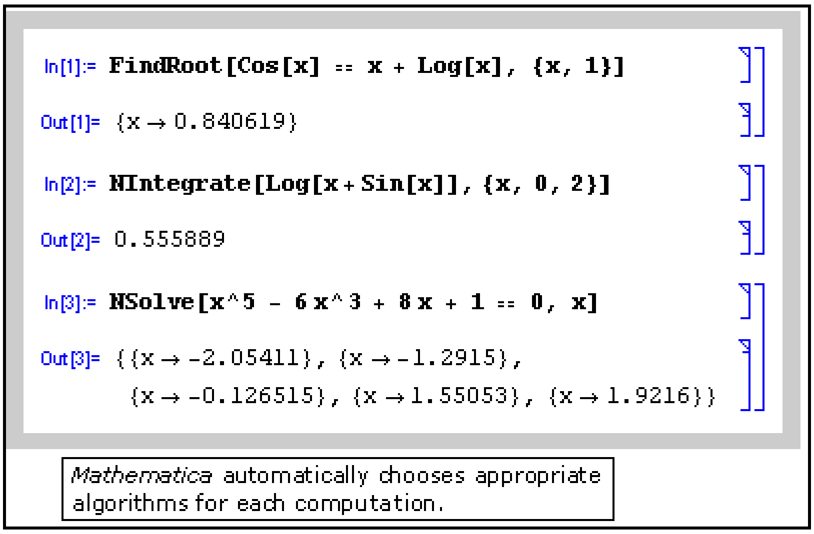
The Mathematica software [http://www.wolfram.com/products/mathematica/tour/page2.html] is very useful at computing equations quickly and accurately. The software can understand the mathematical notation and is able to manipulate complex formulae. For example, the image below shows how Mathematica can integrate a simple function. The requirement for software to easily cope with mathematical notation, both in display and calculation is very important, without such possibilities an example such as this would be much more difficult to undertake.



Mathematica also has the ability to graph both 2D and 3D functions, something that would be very time consuming for a person to do by hand. The ability to draw graphs is maybe not so important at present within the current project, but may become a factor at later stages or within different areas of testing, for example graphs or geometry.

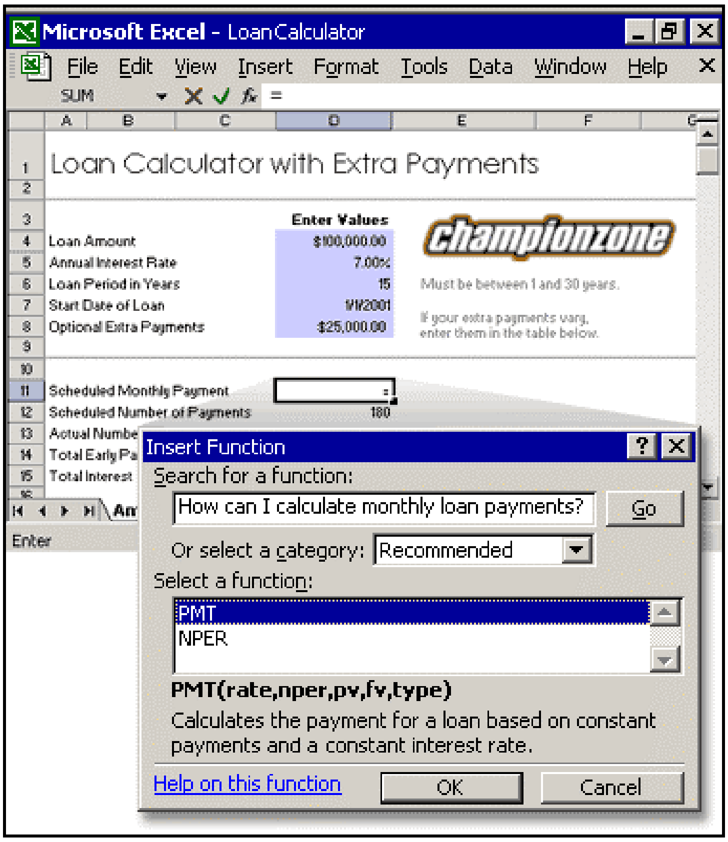


Mathematica has a collection of Algorithms in built with the software. The software chooses applicable algorithms to use in the calculation of results.

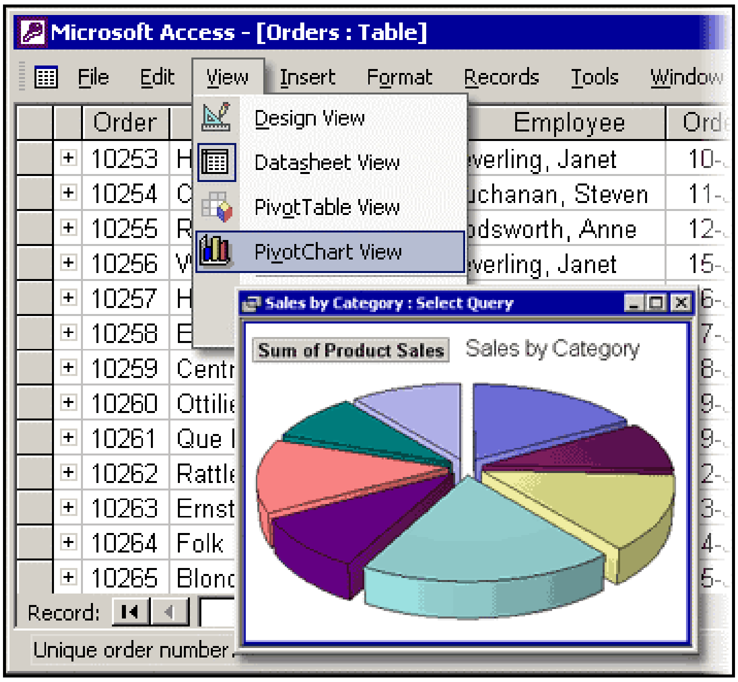


The use of Algorithms is an important factor that will need to be considered when constructing the libraries of questions. For instance, in the displaying of fractions, the lowest factorised fraction should be displayed. E.g. it is better to display $1/3$ than $20/60$. In the situation of recognising results, investigation needs to be made into the problem of un-cancelled fractions.

Microsoft packages, Excel and Access, provide other important mathematical features. Excel [http://www.microsoft.com/office/excel/evaluation/tour/page4.htm] contains a bank of functions that can be applied to large amounts of data.



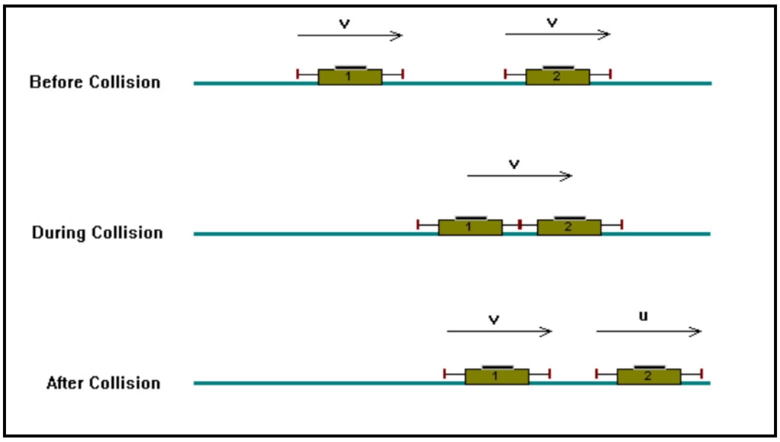
MS Access [http://www.microsoft.com/office/access/evaluation/tour/default.htm] database software can manipulate and organise large amounts of information, this can be done very efficiently. High speed calculation is another factor that needs to be implemented within the teaching libraries, it would be difficult to have an effective teaching aid if the speed was very slow; the student would soon become bored and give up.

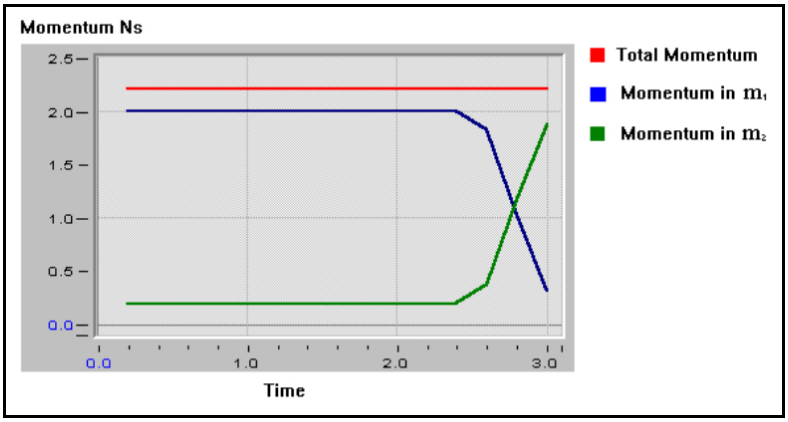


2.2.2 Virtual Experiment Packages

Albert Physics

Albert Physics is a package that examines physical experiments via the use of computer. There is a wide range of test that can be undertaken with Albert including the exploration of pendulums and collisions. The user is able to interact with the software by varying the levels of the factors involved and then watching their effect on the experiment. During and following the research the user is able to watch the experiment on screen and gather data. The screenshot below shows a collision experiment of two bodies, the user is here able to interact and alter the initial speed of each body as well as their masses. Graphs of the experiment can also be drawn to aid understanding the experiment and the physical laws that underpin them.





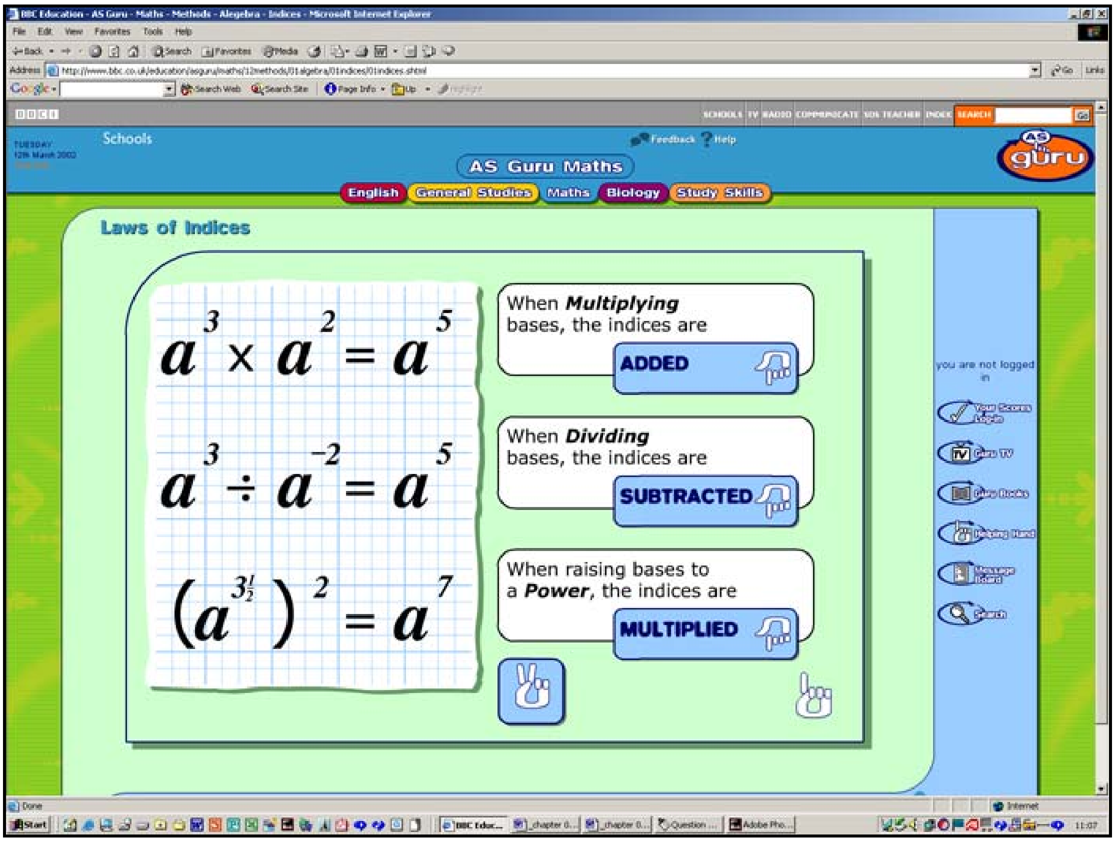
2.2.3 Learning and Testing Packages

Computer Aided Learning is widely used in the discipline of Mathematics. There are many varieties of CAL software and teaching resources, ranging from static pages for students to read through to interactive teaching and assessment tools.

Jill Martin states in the report "Development of tests for Mathletics" [Jill Martin (2001)] that in most cases of CAL resources reviewed that there was a lack of random questions and detailed feedback given to the student in response to their submitted answer.

BBC AS Guru

The AS Guru pages [http://www.bbc.co.uk/education/asguru/maths/] within the BBC Internet site provide a broad range of subject matter to cover the A Level teaching syllabus. AS Guru provides mostly only static teaching and revision pages for each topic, the site does not provide any opportunity for testing and hence the lack of any possibility for feedback if the student is doing anything wrong. The screenshot of AS Guru shows an Indices tutorial page where the user selects a choice of numbers and then the site explains the rules of indices. The page then clarifies the rules, i.e. "When dividing bases, the indices are subtracted".



www.sosmath.com

This Internet site [www.sosmath.com], like the BBC's Guru, provides static teaching pages. 'Cyberexams' are also available across a broad range of subjects, however the questions have limited variation. The questions appear to be mostly of the multiple-choice variety, and the algebra pieces in the questions are created from pictures, removing any option for random numbers, hence the questions in the tests will be identical on each attempt of the exam. The obvious danger here is that the student will learn the answer by continually returning to the same question rather than understanding the principals that underpin the material being tested.

www.mathsnet.net

Like the AS Guru and SOS Maths sites, this web based teaching tool [www.mathsnet] aims to provide the student with an understanding of mathematics across the A Level syllabus. Mathsnet uses the WebEQ viewer, which allows the user to view mathematical notation in its proper form without displaying it in the form of pictures. The most useful tutorials guide the student by explaining processes in a step-by-step fashion. For instance in a bracket expansion, the computer will show each step and indicate to the user exactly where each coefficient is coming from. This site takes advantage of mathematical equations, a graphic calculator and other Java applets in order to instruct the student in a way that is interactive and beneficial. The Mathsnet pages do not provide students opportunity for self-assessment.

Maths Online Gallery [http://www.univie.ac.at/future.media/moe/]

This comprehensive website provides teaching aids and testing facilities. The site provides vast opportunities for interaction during learning, ranging from definitions of sets through to a function plotter. The simple 'drag and drop' system for selecting answers, although sometimes reducing options of question types provides a logical process of indicating the chosen answer.

Test and Learn (TAL) Mathematics at Bristol University

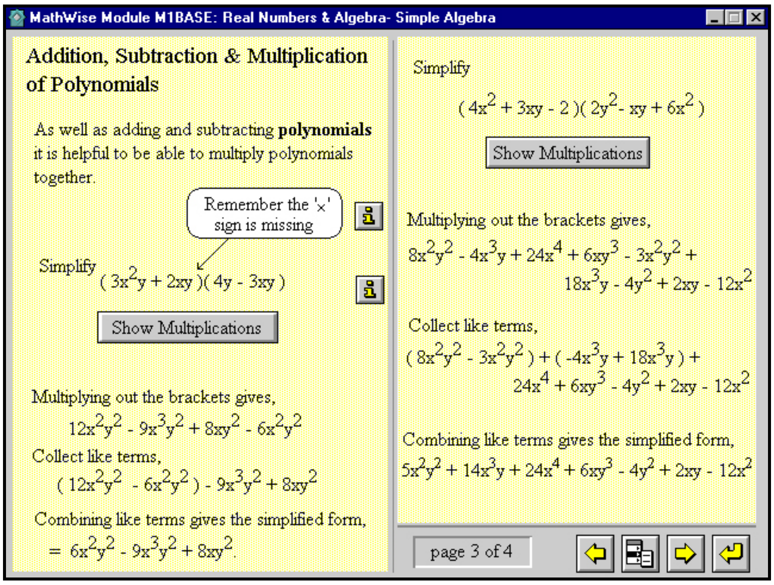
Bristol university have built an online testing suite for mathematics because research found that standards of mathematical skill and understanding are difficult to maintain and also that this aids in encouraging students to test their skill and understanding is one way to help them. Testing is done via login and tests are done in under one of two conditions, either supervised tests where all students do the same test at the same time, or students take a named test in their own time (between 0830 and 1730hrs), with the named test they get a randomly chosen test and never get the same test again. Users can also login via submitting an email only and then taking a test. Teachers can choose a selection of questions and create their own tests for the students to undertake. [http://www.tal.bris.ac.uk/Maths.htm]

Mathwise

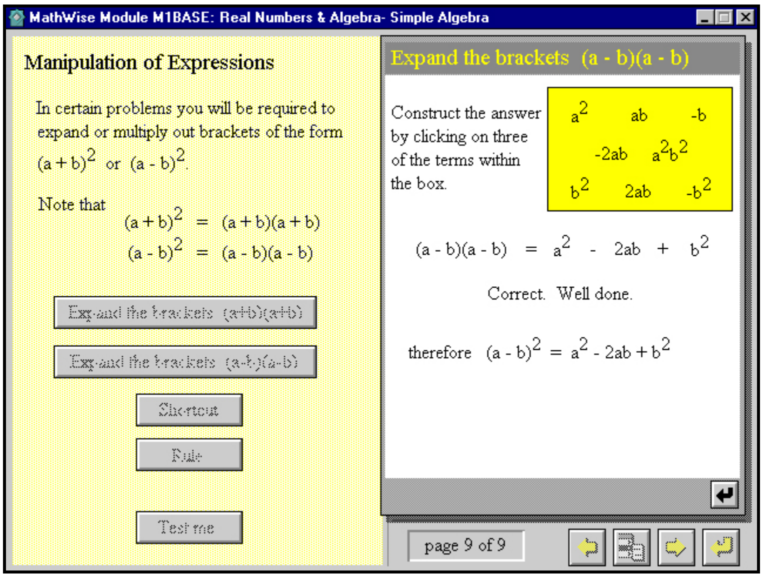
Mathwise is an integrated learning environment for teaching undergraduate mathematics. It comprises of mathematical and application modules, reference material, assessments and resource tools. Mathwise has four main sections to the assessment part of the program in order to aid the teaching process [http://www.bham.ac.uk/mathwise/]:

- Examination mode - no visible feedback is displayed on the screen, this can be used to grade students;
- Practice mode - marking is given at the end of each question;
- Learning mode - feedback is displayed at the end of each part of the question;
- Help mode - feedback at the end of each part and the option to reveal answers.

Mathwise provides comprehensive teaching of mathematical material and via explanations and animations the student easily gains understanding of the subject matter. The screenshot below shows the expansion of polynomials as it comprehensively goes through the processes of multiplying out the brackets and then collecting the like terms together. Via highlighting each term systematically the student can see exactly where each term has come from within the expansion.



Another feature within Mathwise is student interaction, a question is displayed and the student has to click on the parts of the equation they think makes up the solution. Feedback to the student reveals how they have done.



As Greenhow (1998) reports, there is a lack of variation within these questions and sometimes the material presented and feedback given is insufficient and weak. Most questions are completely fixed either via the use of graphics or by the constant display of the same mathematical information. This reduces the number of questions on offer to the student, the main ramification of this is that if the student returns to Mathwise, or indeed any software with the lack of randomisation, they are eventually faced with exactly the same questions and information they have seen before.

Aim

AIM (Alice Interactive Mathematics) is a web-based system designed to administer graded tests with mathematical content. The emphasis of AIM is on formative assessment. AIM is mostly written with the programming language MAPLE, hence AIM can take full advantage of the mathematical material built into MAPLE. AIM takes advantage of using MAPLE syntax in the submission of answers, most questions require the answer inputted as free text. For example $\sin^3(x)/(3\cos(x))$ would be submitted as sin(x)^3/(3*cos(x)). Strickland (2002) states, "It seems clear that questions with this flexibility are pedagogically more valuable than multiple choice or multiple response questions." A downside to this however is the need for students to learn this syntax in order to input their answer. Several key advantages seen within AIM are:

- If a student gives a correct answer that differs from that supplied by the teachers then AIM can still recognise this, for example accepting 2*x instead 2x.
- If a student incorrectly calculates a system of equations then AIM can process the equations with the users incorrect solutions and show that they don't work out.
- Questions that have more common errors can be linked in with more detailed feedback to the student. In the example of the integration of $\sin^2(x)$, AIM can examine the integrand and recognise that students are tempted to give common answers of $\sin^3(x)/3$ or $\sin^3(x)/(3\cos(x))$. AIM automatically generates an explanation of the error to the student if one of these results occurs.

[http://allserv.rug.ac.be/~nvdbergh/aim/docs/] [Strickland (2002)]

The main features of AIM include the use of MAPLE as the engine for test delivery. This makes it easy to design, display and automatically grade questions with randomisation. A typical example is a question, which asks the student to compute a symbolic integral of a randomly chosen product of trigonometric functions and exponentials, the system then verifies whether their answer is correct through differentiation. The tutor can easily include mathematical formulas and MAPLE graphics are generated at run-time. There is also the implementation of several automatic methods of giving partial credits, feedback and penalties for wrong answers, these include the use of hints and sub questions that, if the student decides to views them, generate a penalty. Predefined question types include algebraic, matrix, constant, multiple response, multiple choice, string. In addition any MAPLE type can also be used. Grade reporting and monitoring capabilities, including the ability to collect surveys electronically. [http://allserv.rug.ac.be/~nvdbergh/aim/docs/]

After a trial at Sheffield University, students had mixed feelings with the AIM system. On the positive side users liked the fact that questions could be re-tried if the initial answer was incorrect, the instant feedback following a question was also seen as popular. Another plus for AIM usage was the fact that students didn't need to be on campus or in a computer room. The questions could be answered remotely from home and questions could also be done in their own time. However the lack of marks given for method and the lack of partial credit given to students in response to their solutions were seen to be negative factors to the use of AIM. [Strickland (2002)]

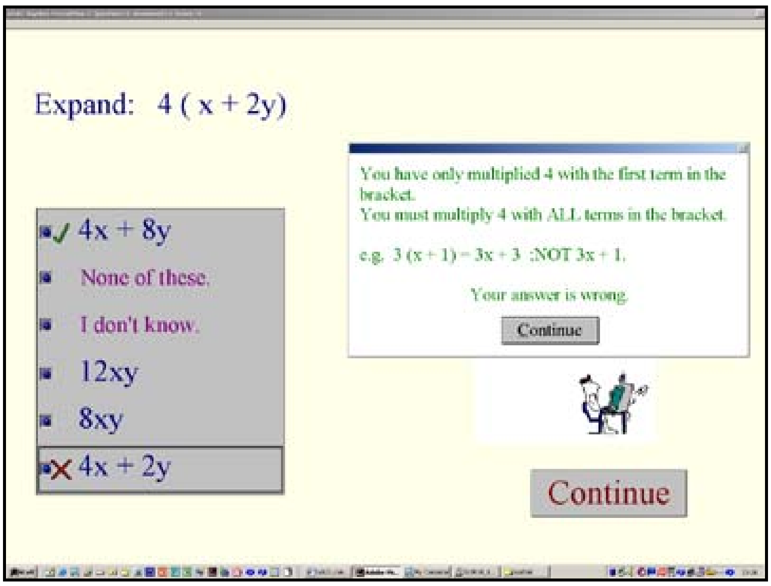
Mathletics

Mathletics, created with Question Mark Designer, is an online teaching and assessment package designed by Dr. Martin Greenhow of Brunel University. The package is widely used throughout Brunel and universities across the country to test students' understanding of mathematical material.

Mathletics now comprises some 4500 questions spanning 175 different mathematical skill areas within the GCSE to level 1 university attainment. Increasingly, Mathletics has become an integral part of teaching at Brunel University; in the academic year 2000/2001 some 600 foundations; mathematics, engineering and biological sciences students took over 23,000 diagnostic and continual assessment tests. [Greenhow (2002)]

Within Mathletics are a series of tests called the 'Algebra Pentathlons', each consisting of five assessments that cover the knowledge of algebraic mathematics. This project will undertake the development of several areas of Mathematics covered within the Mathletics Algebra Pentathlon, namely: Expanding Brackets and Indices. These new question libraries will be created within Perception, a new software package from Question Mark.

The screenshot of Mathletics shows a sample question taken from the Algebra section. The Question is posed and then alternative results are provided, the student selects the correct answer and then clicks the Continue button to reveal the answer. Although randomly chosen from a bank there is still have a limited number of questions to call upon.



In the above expanding bracket question the student is asked to expand $4(x+2y)$. The student is provided with six alternative results from which to choose their answer. The alternatives include "None of these" and "I don't know". These also have their importance as possible answers. Sometimes the correct result is not displayed and in this circumstance "None of these" is the correct result. This is done in order to stop students automatically knowing the result is one of those being displayed and perhaps doing the question backwards. Meanwhile when the student selects "I don't know" it shows the tutor that they seriously don't know the result and that they are simply not guessing; sometimes in this situation the computer provides a hint and allows a re-try.

Each of the possibilities will provide feedback to the student when selected. This feedback will initially indicate whether their selection is correct or not and then secondly either confirm the rule if correct or instruct the student where they went wrong if their chosen solution was incorrect. The clicking of "I don't know" will display feedback where the appropriate rules are restated. For example, in the above question expanding $4(x+2y)$, the student incorrectly selects $4x+2y$, the feedback displayed explains that their result was wrong, highlights the mistake they have made and then restates the rules of expanding brackets. *"You have only multiplied the 4 with the first term in the bracket. You must multiply the 4 by ALL terms in the bracket. E.g.* $(3(x-1)=3x-3$ *NOT* $3x-1$. *Your answer is wrong."*

Mathletics stores answer files from the tests so that tutors can follow student and class progress and gain insight into their learning and understanding of the material.

2.3 The importance of Objective Tests

Diagnostic tests are possible via question papers, but the process is time consuming in the marking process for the tutor. Having the ability to carry out these tests via a computer provides great advantages including the possibility for students to continually look at and retake tests to improve their understanding. Instant marking and feedback surely offers a different "quality" of learning, therefore it is different in a fundamental way.

By creating question libraries of varying mathematical subject matter, both teachers and students can use them to test a wide variety of knowledge and also broaden the scope of the student's understanding. The tests could be used to analyse the prior knowledge of a group of students with the express aim of gaining information via the answer files to allow the students to be brought to the same level of insight before commencing a new course. Alternatively the tests could be used half way through or at the end of a module to gauge the development of students. This analysis of progress can be done for individuals and the class as a whole.

Research at Keele University [http://www.keele.ac.uk/depts/ma/diagnostic/] highlights the importance of diagnostic testing, especially within the field of mathematics. The desirability of assessing the current mathematical ability of students on entry to any course is self-evident. In addition, the variety of different examinations, assorted mathematical backgrounds, (including access and mature students), will reinforce these demands in order to help students achieve a common core of mathematical skills. Furthermore, such testing will provide data on the long-term trend of a typical student's actual capabilities and any associated course management and resource implications. The research at Keele also highlights another advantage of the tests, that the primary aim of diagnostic testing is to help students to achieve their full potential. Moreover, the gathering of national information regarding the mathematical abilities of students in Higher Education will also be possible. Such information will help long term in planning in HE and also in informing the A-Level curricula. The impact of students dropping maths after AS level also needs consideration; last year (2001) 29% of students failed AS level mathematics (this figure was previously 16%).

Work done at Nottingham University [http://www.nottingham.ac.uk/cal/mathtest/mathmain.htm] in computer aided learning research lists several benefits of the tests:

- Encouraging students to be self-critical
- Allowing students to obtain immediate and individual feedback
- Giving students their own learning profile
- Allowing students to direct their own additional studies

Greenhow (2002) suggests that there are several key advantages in using diagnostic tests. Reasons for using online diagnostic tests therefore include:

- Revision of GCSE and A-level material and retry of the tests. Even if the answer files were not looked at, this aspect would mean that diagnostic tests would still be worth doing.
- Individual diagnostics on a student-by-student basis. Individual profiles are emailed to all students and tutors. This results in a clear action plan for many students who “hit the ground running” when their courses start.
- Formation of a whole class view, whereby bridging sessions can be planned.
- Staff appreciate the automatic marking and feedback, since it leaves them free to do what they do well i.e. talking and listening to students

Paper based allow the testing of a students’ understanding of a mathematical procedure, but diagnostic testing via a computer allows the testing of individual skills that make up these actions. The following question is taken from the Brunel University, Foundations of Science, 2002 Basic Mathematical Methods Exam:

Express $\frac{(3+\sqrt{2})}{(3-\sqrt{2})}$ in a form where the denominator has no surds.

A student needs to have a grasp of several key skills in order to answer this question. An understanding of the method of solution i.e. multiplication by the conjugate is obviously required, but knowledge of expanding brackets and the rules of indices is also needed. Hence it can be seen that a relatively simple problem can be broken down into smaller skills that the student needs to understand prior to being able to solve the question.

$$\frac{(3+\sqrt{2})}{(3-\sqrt{2})} = \frac{(3+\sqrt{2})}{(3-\sqrt{2})}\times\frac{(3+\sqrt{2})}{(3+\sqrt{2})} = \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{(11+6\sqrt{2})}{7}$$

The importance of identifying assumed skills and tested skills are fundamental to the setter of objective questions. The subject of skills, both assumed and tested will be examined in later chapters.

Chapter Conclusions

This chapter has shown that Computer-Aided Learning has many guises even within the field of Mathematics. There are many different uses that computers offer in mathematics, whether in calculation or in teaching. Obviously each Computer-Aided Learning product has its own aim, but generally each of the types of software available has advantages and disadvantages in what they provide, some have a greater scope with quick and accurate calculation whilst others have serious drawbacks in their ability to display and test with random questions.

This chapter has also revealed the advantages of teaching and running diagnostic tests via computers. Investigations have pointed to the software available and the need for questions to test individual skills, to be randomised, to be easily displayed, and then following the questions feedback to be given in explanation to the result submitted.